A Model of External Burning of Liquid Fuels

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Injection of a pyrophoric liquid fuel into the airstream about a vehicle can generate control forces on the vehicle. A method for calculating the performance of such a system is described. Particular attention is paid to the evolution of the drop cloud resulting from breakup of the liquid jet and to diffusion and combustion of the vapor resulting from drop evaporation. Combustion is calculated assuming local equilibrium, with its rate controlled by evaporation.

I. Introduction

INJECTION of a liquid pyrophoric fuel into a supersonic airstream produces a high-pressure region locally on the surface. This concept, which can be used to achieve attitude control, is known as external burning. The evaporating and burning fuel expands and deflects a portion of the airflow outward from the vehicle surface. Both the region within the combustion zone and the region between the resulting shock and burning volume represent areas of high pressure that produce forces available for vehicle control. Figure 1 shows one possible application of external burning.

To design an external burning system, an understanding of the burning process is necessary to develop a method for computing the force. Two approaches are possible. One could develop a model primarly for computing the force based on approximating the gross features of the process relying on experimental data to obtain correlation. The main difficulty with this approach is that at present there is insufficient experimental data. A second approach describes the details of the flow starting from first principles and then, using a minimum number of correlations based on experiment, determines the pressure distribution as a function of distance from the injector, and specific impulse. The latter approach is the subject of this paper.

II. Physical Basis and Assumptions for the Model

When a liquid is injected into a supersonic airstream, the liquid jet is diverted in the direction of airflow and broken into a spray of small droplets as its progresses in the flow direction. The presence of the jet and spray generates a shock in the airflow. Within the envelope of this shock, some of the air is diverted past the jet spray, and some flows through the spray and carries the droplets downstream. If the injectant is pyrophoric, combustion of the fuel vapor-air mixture starts and the droplets continue to evaporate and burn while moving downstream. To develop a model for this type of flow phenomenon requires the solution of many problems, some of which may be solved separately from the overall flow process: for example, problems of the jet penetration and spreading, drop formation, droplet evaporation, and jet shock envelope. An analysis of this type requires certain assumptions so that the problem may be solved by presently available analytic and computational techniques. Some of these assumptions are now given to indicate some limitations placed on the model.

1) The inner flow (jet breakup and combustion) is based on averaged properties of the flow, whereas the shock layer flow downstream of the jet breakup region is assumed to be axially symmetric.

2) Combustion does not changes the spray envelope in the proximity of the injector.

3) Boundary-layer effects are neglected.

4) The mass loss from the jet is based on Clark's data.[1] The upper-limit distribution is assumed for predicting the number of drops as a function of size.

5) Reaction rate is assumed to be infinite, i.e., the evaporation rate equals the combustion rate, and the inner flow is assumed well mixed.

III. Conservation Equations

The model of the generalized flowfield (Fig. 1) is divided into two major regions to simplify the necessary computations: the jet shock layer and the jet spray envelope. The jet spray envelope is further divided into jet breakup and combustion zones; combustion occurs in both zones. The jet breakup zone is made up of the jet spray envelope from the injector to a point downstream where the jet is completely broken into droplets; the combustion zone accounts for the flow from the end of the breakup zone to the end of the vehicle.

In the jet breakup zone, the air entering the model and the amount of fuel stripped from the jet are functions of distance from the injector. This model accounts for these variations and uses an equilibrium thermochemistry program as a subroutine to make all the necessary combustion calculations throughout the spray envelope.

Conservation of Mass

The calculation model consists of air flowing into the model, liquid droplets ripped from the jet, and vapor resulting from evaporation of the liquid droplets. The conservation of mass requires that the mass of air flowing into the model plus the mass of the droplets stripped from the jet must equal the mass of air flowing out of the model, plus the mass of evaporated droplets. In differential form this can be expressed as

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\rho UA + \rho_\ell U_\ell A\right) = 0 \tag{1}$$

where subscript ℓ refers to liquid.

Conservation of Momentum

The momentum and force balance requires that (rate of momentum in) − (rate of momentum out) + (sum of forces on the system) = 0. In differential form the momentum equation becomes

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\rho U^2 A + \rho_\ell U_\ell^2 A\right) = A\frac{\mathrm{d}P}{\mathrm{d}x} \tag{2}$$

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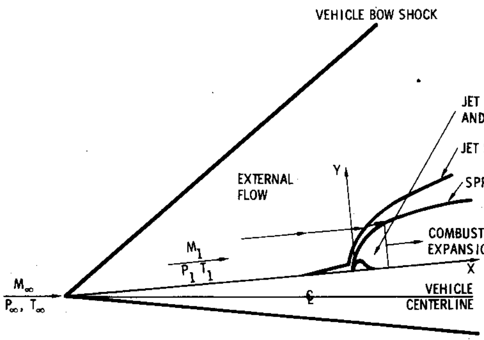

Fig. 1 External burning flowfield.

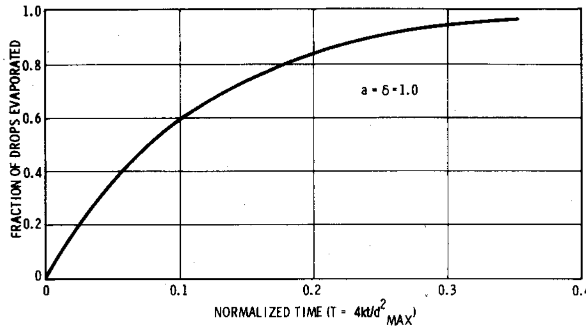

Fig. 2 Evaporation as a function of nondimensional time.

Conservation of Energy

From conservation of energy, the total enthalpy and kinetic energy transported into the calculation zone by the air and jet droplets and vapor must equal the enthalpy and kinetic energy leaving the zone. In this case, the enthalpy includes the heats of formation of all species considered, i.e.,

$$\frac{d}{dx}\left[\dot{m}\left(h+\frac{1}{2}U^2\right)+\dot{m}_\ell\left(h_\ell+\frac{1}{2}U_\ell^2\right)\right]=0 \tag{3}$$

where

$$h=\sum_{i=1}^{n}\left(x_i\Delta h^0_{f_i}+\int_{T_0}^{T}x_iC_{p_i}\,dT\right) \tag{4}$$

IV. Evaporation

Drop Evolution

Droplet evaporation was computed in this model using the upper-limit distribution function. From experimental and analytic studies,[2,3] this particular distribution function was found to be most realistic for prediction of the vaporation of a cloud of droplets as a function of maximum drop size and droplet stay time. The distribution function can be written in terms of the nondimensional drop diameter $\xi = d/d_{max}$ as

$$N(\xi)=\frac{\delta\exp\left\{-\left[\delta\ln\left(\frac{\alpha\xi}{1-\xi}\right)\right]^2\right\}}{\sqrt{\pi}\,\xi^4(1-\xi)} \tag{5}$$

The volume fraction evaporated as a function of time is

$$\gamma(t)=1-[V(t)/V(0)]$$

where $V(t)$ is the volume of the droplets at time t, and

$$\frac{V(t)}{V(0)}=\frac{\int_0^{\xi_{max}}\xi^3N(\xi)\,d\xi}{\int_0^1\xi_o^3N(\xi_o)\,d\xi_o} \tag{6}$$

The individual drops are assumed to burn by the area law

$$d(d/2)^2/dt=-k \tag{7}$$

$$d^2=d_o^2-4kt \tag{8}$$

Dividing by d^2_{max} provides a relation for ξ

$$\xi=\left(\xi_0-4kt/d^2_{max}\right)^{1/2} \tag{9}$$

By noting that drops initially the same size evaporate at the same rate, then

$$N(\xi)\,d\xi=N(\xi_o)\,d\xi_o \tag{10}$$

A suitable form for $V(t)/V(0)$ can now be obtained by substituting the above relations into Eq. (6); then

$$\gamma(T)=1-[V(T)/V(0)]$$

$$=1-\frac{\delta}{\sqrt{\pi}}\int_{\sqrt{T}}^{1}\frac{\left(\xi_o^2-T\right)\exp\left\{-\left[\delta\ln\left(\frac{a\xi_o}{1-\xi_o}\right)\right]^2\right\}d\xi_o}{\xi^4(1-\xi_o)} \tag{11}$$

where

$$T=4kt/d^2_{max}$$

A plot of $\gamma(T)$ is shown in Fig. 2.

The stay time of the particles t is obtained from the constant-lag law between two station increments i and $i+1$. The nondimensionalized time T as defined above can then be obtained as a function of stay time, burning rate constant, and maximum drop size. The shape parameters a and δ are given by

$$\delta=\left[\frac{\left(\bar{d}_v/d_{max}-\frac{1}{2}\right)-\left(\frac{5}{2}\bar{d}_n/d_{max}-2\right)}{\ln\left(d_{max}/d_n-1\right)-\ln\left(d_{max}/d_v-1\right)}\right] \tag{12}$$

$$a=\left(d_{max}/\bar{d}_v-1\right)\exp\left[\left(\bar{d}_v/d_{max}-\frac{1}{2}\right)/\delta^2\right] \tag{13}$$

where $\bar{d}_v$ is the most probable drop size based on volume distribution and $\bar{d}_n$ is the most probable drop size based on number distribution.

The maximum drop size can now be calculated using Eqs. (12) and (13) and previous results[4,5] as follows

$$d_{30}=\frac{48d_{jet}}{W_\ell^{0.375}(R/M)^{0.25}} \tag{14}$$

$$d_{max}=d_{30}\left(1+3ae^{1/4\delta^2}+3a^2e^{1/\delta^2}+a^3e^{9/4\delta^2}\right) \tag{15}$$

Mass Loss From the Jet

The mass stripped from the liquid jet as a function of downstream position is required. It was shown[6] that the fractional amount of mass remaining on the jet could be written as

$$\frac{\dot{m}_j}{\dot{m}_{\ell TOT}}=\frac{\epsilon^2_{min}\sin\alpha_o}{\epsilon^2}\quad\text{for }\epsilon>\epsilon_{min} \tag{16}$$

where

$$\epsilon = \lambda^2 / \overline{q}$$

The mass loss from the jet then becomes

$$\dot{m}_\ell / \dot{m}_{\ell_{TOT}} = 1 - \lambda_{min} / \lambda^4 \sin \alpha_o \quad \text{for } \lambda > \lambda_{min} \tag{17}$$

For $\lambda > \lambda_{min}$, no mass is assumed to be lost from the jet. The value for λ_{min} was determined to be

$$\lambda_{min} = 1.36 \overline{q}^{\,0.581} \tag{18}$$

V. Solution for the Jet Breakup Zone

The jet breakup zone is generally quite asymmetric, with penetration three or more times greater than spreading as defined by the jet spray envelope. The conservation equations are solved in this zone using a series of control volumes, as shown in Fig. 3. The required geometry of each control volume is obtained by using empirical fits for the jet shock and spray envelope.[6] It is assumed, in the absence of information on the effective permeability of the jet spray envelope, that shock-layer streamlines are straight and maintain the direction at which they leave the jet shock.

VI. Solution for the Combustion Zone

Most of the combustion and force generation occurs downstream of the jet breakup zone. The calculation proceeds stepwise downstream, and at each step the inner and outer flows must be calculated, and direction and pressure matched at the boundary. The general calculational procedure is illustrated in Fig. 4. Given a dividing streamline, the calculation of the outer flow is the same as calculating the shock layer about a solid body.

A convenient method to calculate the shock and the properties of the flow between the shock and body is given by Maslen.[7] This is an inverse method and therefore requires an iterative technique to solve if the shock is to be generated from a given body. The procedure assumed an initial estimate of the shock in the nose region and takes small variations of this estimate until the flow equations are satisfied. The solution is then continued downstream by iteration on the value of the shock slope. The shock position and curvature are obtained by simple numerical integrations. The conditions at the shock are obtained from the oblique shock relationships. Because the entropy is constant along the streamlines and is known at the shock, the remaining thermodynamic properties follow after the pressure is specified. The main assumptions are that the flow is parallel to the shock and that the flow is axisymmetric.

VII. Downstream Mixing

Calculation of the inner flow in each streamwise increment is done using the conservation equations and allowing for the downstream evolution of the drop distribution, as in the jet breakup zone. As the calculation moves downstream, it is

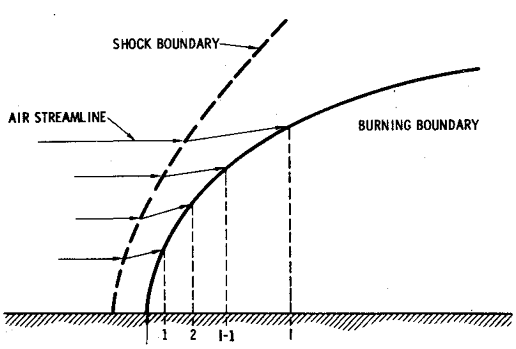

Fig. 3 Typical upstream control volumes.

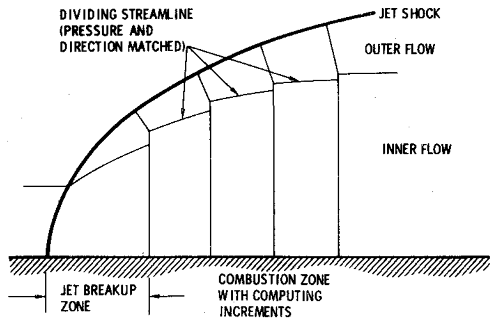

Fig. 4 Structure of downstream combustion zone.

necessary to account for air addition to the inner flow by mixing, as the drop cloud diffuses outward. In doing so, it is assumed that at the beginning of the downstream region the jet consists of a drop cloud that evaporates as it convects downstream and that the evolved vapor spreads laterally by turbulent diffusion, the vapor from each point in the drop cloud spreading according to empiric fits to gas jet data. Provision is made in the analysis for lateral diffusion of the drop cloud itself, although no really reliable data on this subject seem to exist, because some such diffusion is likely to occur as a result of the outward flow of vapor generated by phase change within the cloud.

We therefore consider a drop cloud that originates at a point and diffuses radially as it convects downstream, so that its mass flux distribution is proportional to $\exp - (r/\sigma_\ell)^2$, where σ_ℓ characterizes the width of the drop (liquid) distribution and is linear with downstream distance x, and r is radial distance from the center of the distribution.

Now each point of this distribution is a source of vapor, which itself convects downstream as a Gaussian distribution, following the jet-mixing data of Ref. 8. Lacking information on the spatial distribution of drop sizes, we assume that they are uniformly distributed. Then the source strength at any point is proportional to the liquid density there. Denoting source strength by $S(r,x)$, we have

$$S(r,x) = S_o(x) \exp - (r/\alpha_\ell)^2 \tag{19}$$

To find the x dependence of the source strength, we must consider the rate of evaporation of the drops. Integrating Eq. (19) over all r, and accounting for θ symmetry, we have

$$\int_{all\, r,\theta} S_o \exp - (r/\alpha_\ell)^2 r \, dr \, d\theta = -\dot{m}_j \frac{d}{dx} \frac{V(x)}{V(0)} \tag{20}$$

Here the term on the right expresses the fact that the source strength at any plane x = constant is equal to the jet flow rate times the proprotional rate of evaporation (V is volume of liquid) at that plane. Performing the integration in Eq. (20), we obtain

$$S_o = \frac{\dot{m}_j}{\pi \alpha_\ell^2} \frac{d}{dx} \frac{V(x)}{V(0)} \tag{21}$$

The volume of the remaining drops is given by Eq. (11) above. Transforming and differentiating that expression with respect to z yields

$$\frac{d}{dx} \frac{V(x)}{V(0)} = -\frac{3}{2\sqrt{\pi}} \frac{\delta k}{r_{max}^2 \langle u_\ell \rangle} \int_{\xi_{min}}^{1} \left(1 - \frac{kx}{r_{max}^2 \langle u_\ell \rangle \xi^2}\right)^{1/2} \times \frac{\exp - \left(\delta \ln \frac{a\xi}{1-\xi}\right)^2}{\xi^3 (1-\xi)} d\xi \tag{22}$$

where

$$\xi_{\min} = \left(kx / r_{\max}^2 < u_\ell > \right)^{1/2} \tag{23}$$

Here a, δ, and $r_{\max}$ are parameters of the assumed Upper-Limit Distribution [Eqs. (12) and (14) above]; $<u_\ell>$ is the average drop velocity, and k is the rate constant for evaporation, defined as the time derivative of the square of drop radius, assumed independent of drop radius.

Now substitution of Eqs. (21) and (22) into (19) gives a complete representation of source density as a function of r and x. The vapor mass flux at each point (r,x) consists of contributions from all points (r',x') of nonzero source density

$$\dot{m}_v(r,x) = \int_{z'=0}^{z} - \frac{\dot{m}_j}{\pi^2 \sigma_\ell^2 \sigma_v^2} \frac{d}{dx'} \frac{V(x')}{V(0)} \times \int_{z',y'=-\infty}^{+\infty} \exp\left\{ - \left[\left(\frac{y-y'}{\sigma_v} \right)^2 + \left(\frac{y'}{\sigma_\ell} \right)^2 + \left(\frac{z-z'}{\sigma_v} \right)^2 + \left(\frac{z'}{\sigma_\ell} \right)^2 \right] \right\} \cdot dx' dy' dz' \tag{24}$$

Now the σs are functions of x. For linear growth

$$\sigma_\ell = a_\ell x' \tag{25a}$$

$$\sigma_x = a_v (x - x') \tag{25b}$$

and completing the square in the second integral of Eq. (24) yields

$$\dot{m}_v(r,x) = -\frac{\dot{m}_j}{\pi} \int_{z'=0}^{z} \frac{d}{dx} \frac{V(x')}{V(0)} \frac{dx'}{a_\ell^2 x'^2 + a_v^2 (x-x')^2} \times \exp - \frac{r^2}{a_\ell^2 x'^2 + a_v^2 (x-x')^2} \tag{26}$$

Thus vapor mass flux is found by numerical integrations of Eq. (22) and (26).

Some results are shown in Fig. 5, for two different values of $<u_\ell>$. These results suggest a useful simplification. The results look Gaussian. This is to be expected on a general consideration, since Eq. (24) is really a convolution integral, and the convolution of a Gaussian is Gaussian. Although this is not necessarily true for an arbitrary distribution of sources, such as described by Eq. (22), for this case it is in fact a quite accurate approximation.

This fact allows a substantial simplification for two reasons: first, Eq. (26) need only be integrated at any x for two values of r, one for each parameter in the Gaussian; second, the outward stepping of the dividing streamline is readily controlled by a Gaussian. The first of these steps assumes

$$\dot{m}_v(r,x) = \dot{m}_v(0,x) e^{-(r/\sigma)^2} \tag{27}$$

where σ is the variance for the integrated vapor profile. As the notation of Eq. (27) implies, the central value $\dot{m}_v(0,x) \equiv \dot{m}_{vo}$ is found by integrating Eq. (26) at $r=0$. For convenience, σ is found by finding $m_v(1,x) \equiv \dot{m}_{vl}$ from which Eq. (27) yields

$$\sigma^{-2} = \ln \dot{m}_{v0} / \dot{m}_{vl} \tag{28}$$

Now with a Gaussian profile, no radius, however large, can contain all the injectant. Thus a cutoff is chosen so that a fraction C_{crit}, which ought to be nearly one, is contained. The radius R to contain a fraction C_{crit} of the jet is defined by

$$C_{\text{crit}} = \frac{\int_{r=0}^{R} \dot{m}_v(r,x) r dr}{\int_{r=0}^{\infty} \dot{m}_v(r,x) r dr} \tag{29}$$

which with Eq. (27) and (28) is reducible to

$$R = -\ln(1 - C_{\text{crit}}) / \ln(\dot{m}_{v0} / \dot{m}_{vl}) \tag{30}$$

This is the height of the C_{crit} point above the jet centerline in the symmetry plane. The height above the surface is

$$R(z) + H \tag{31}$$

where H is the equivalent axisymmetric EB jet penetration, defined as the radius of the semi-circle of equal area, at the entrance of the downstream region.

At this point it is necessary to find the increase in the inner flow cross section resulting from Eq. (31). As essentially all the jet is in the form of droplets at $x=1$, the drop cloud is assumed uniformly distributed over the inner flow region at $x=1$ and thereafter. Then δr equals the change in R; for example, between two points x_i and x_{i+1}, $\delta r = R(x_{i+1}) - R(x_i)$. This directly gives the outward stepping of the dividing streamline in the downstream region of an EB flowfield, as controlled by vapor evolution.

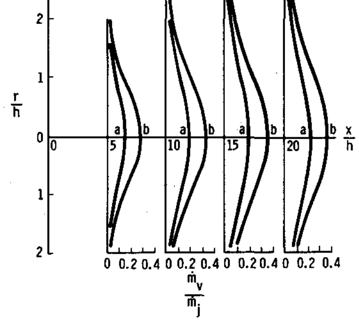

Fig. 5 Vapor mass flux profiles evolved from a liquid jet.

VIII. Remarks and Conclusions

The present model of external burning of liquid fuels is intended to correspond reasonably well to physical reality. It includes treatment of liquid jet breakup, the evolution by evaporation of the drop cloud thus produced, the lateral diffusion of the resulting cloud of drops and vapor, and the interactions of these processes with the combustion of the liquid fuel. The jet shock and the process of combustion and evaporation are calculated together, in a coupled manner, as the calculation advances downstream. Two phenomena not included in the model are chemical kinetics and flow separation. As to the former, the computational difficulties of inclusion are very large, and in many cases the use of evaporation as the rate-limiting process is a reasonable procedure. As to flow separation, because penetration of liquid jets is typically much greater than their spreading, upstream separation is usually relatively minor in physical extent.

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